EULER'S THEOREM.

The number of faces and summits in any polyhedron taken together exceeds by two the number of its edges.

Proof.

First Case. If all the faces are triangles; then by Descartes's theorem,

$$F = 2(S - 2).$$

But also $2E = 3F$, since each edge belongs to two faces, and so we get a triangle for every time 3 is contained in $2E$. By adding, we have

$$2E = 2F + 2(S - 2),$$

that is,

$$F + S = E + 2.$$

Second Case. If not, all the faces are triangles. Since to any summit go as many faces as edges, we may replace any polygonal face by a pyramidal summit without changing the equality or inequality relation of $F + S$ to $E + 2$; for such replacement only adds the same number to F as to E and changes one face to a summit. But after all polygonal faces have been so replaced, $F + S = E + 2$ by our first case. Therefore the relation is always equality.

[These theorems are substantially one; the second is also due to Descartes, having been published in 1860 in his *Œuvres Inédites.—W. M. T.*]

PROOF OF A PROPOSITION IN MODERN GEOMETRY.

By Mr. R. D. Bohannan, University of Virginia.

A,B are any two points on the curve which is the locus of the intersection of corresponding rays of two homographic ray-systems. If A,B be made the centres of two ray-systems whose corresponding rays intersect on this curve, these two ray-systems are homographic.

The curve which is the locus of corresponding rays of two homographic ray-systems is of the second degree and passes through the two ray-centres. Being of the second degree, it is fixed by fixing on it five points A,B,C,D,E. Take on it any sixth point F. The three rays BC, BD, BE may be taken arbitrarily to correspond to the three rays AC, AD, AE. Suppose the ray BK corresponding to BF does not intersect AF in F, but in K. Then we have two curves of the second degree, one passing through the six points A,B,C,D,E,F and the other passing through the six points A,B,C,D,E,K. But the curves have five points in common. Thus K,F are coincident. Therefore, etc.

If AC, AD, AE, AF, BC, BD, BE, BF are homographic, the six points A, B, C, D, E, F, no three of them being on a straight line, lie on a curve of the second degree. Thus, by the preceding proposition, the rays drawn from any two of the six points A, B, C, D, E, F to the other four are homographic systems.

Chasles proves these propositions in the inverse order to that here given. See *Traité de Géométrie Supérieure*, Chapter XXV, Arts. 544, 416.

SOLUTIONS OF EXERCISES.

9

It is assumed that when a gate in a water-pipe is closing the pressure increases uniformly and the discharge decreases uniformly. Investigate an expression for the shortest safe time for closing the gate on the basis of these hypotheses: given the length of the pipe, the velocity of the stream, the working pressure, and the greatest admissible pressure to which the pipe may be exposed.

[*W. M. Thornton.*]

SOLUTION.

Put L for the length of the pipe,
- V velocity of the stream,
- Ω sectional area of the pipe,
- g acceleration due to gravity,
- D heaviness of water,
- p working pressure in the pipe,
- P highest admissible pressure in the pipe,
- T required time in seconds.

The kinetic energy of the stream is

$$\frac{D}{2g} \cdot L\Omega V^2.$$

The velocity and pressure at t seconds after the closing of the valve begins are

$$V_t = V\left[1 - \frac{t}{T}\right], \qquad p_t = p + (P - p)\frac{t}{T};$$

and the work done by the stream in entering this region of high pressure is